

# **The Metro Area Impact of Home Building in Milwaukee, Wisconsin: Comparing Costs to Revenue for Local Governments**



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Housing Policy Department





The comprehensive nature of the NAHB model requires a local area large enough to include the labor and housing market in which the homes are built. The local benefits captured by the model, including revenue generated for local governments, include the ripple impacts of spending and taxes paid by construction workers and new residents, which occur in an economic market area. For a valid comparison, costs should be calculated for the same area.

A local labor and housing market generally corresponds to a Metropolitan Statistical Area (MSA) as defined by the U.S. Office of Management and Budget (OMB). Based on local commuting patterns, OMB has identified the Milwaukee-Waukesha MSA as a metro area consisting of four counties (Milwaukee, Ozaukee, Washington and Waukesha) in the State of Wisconsin (see Figure 1). In this report, wherever the terms local or Milwaukee are used, they refer to the four-county metro area.

### ***Costs Compared to Revenue: Total***

This section summarizes results for both single-family and multifamily construction. Detail by structure type follows, but for many purposes a combined analysis of both types may be most appropriate. Market areas generally require a mix of housing types to accommodate residents of different income levels, different occupations, and who are at different stages in their professional careers. Although it's possible to analyze single-family and multifamily construction separately, such an approach does not reflect the typically integrated character of residential development.

- In the first year, the 100 single-family and 70 multifamily housing units built in Milwaukee result in an estimated
  - **\$6.8 million** in tax and other revenue for local governments,<sup>2</sup>
  - **\$510,000** in current expenditures by local government to provide public services to the net new households at current levels, and
  - **\$3.3 million** in capital investment for new structures and equipment undertaken by local governments

The analysis assumes that local governments finance the capital investment by borrowing at the current municipal bond rate of 3.45 percent.<sup>3</sup>

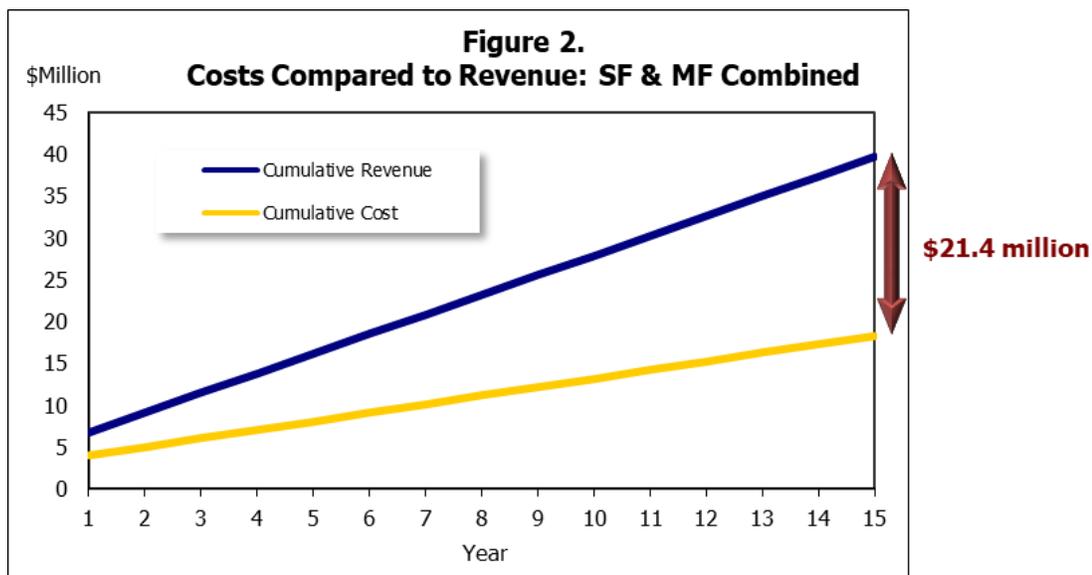
- In a typical year after the first, the single-family and multifamily units result in
  - **\$2.3 million** in tax and other revenue for local governments, and
  - **\$1.0 million** in local government expenditures to continue providing services at current levels

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<sup>2</sup> This assumes that homes are occupied at a constant rate during the year, so that the year captures one-half of the ongoing, annual revenue generated as the result of increased property taxes and the new residents participating in the local economy.

<sup>3</sup> The analysis assumes that there is currently no excess capacity, that local governments invest in capital before the homes are built, and that no fees or other revenue generated by construction activity are available to finance the investment, so that all capital investment at the beginning of the first year is financed by debt. This is a conservative assumption that results in an upper bound estimate on the costs incurred by local governments. The particular interest rate is based on the Bond Buyer Municipal Bond Index, which is based on prices for 40 long-term municipal bonds.

- The difference between government revenue and current expenditures is defined as an “operating surplus.” In this case, the operating surplus generated during the first year is large enough to service and pay off all debt incurred by investing in structures and equipment at the start of the first year by the end of the first year. After that, future operating surpluses will be available to finance other projects or reduce taxes. After 15 years, the homes will generate a cumulative **\$39.7 million in revenue** compared to **\$18.3 million in costs**, including annual current expenses, capital investment, and interest on debt (Figure 2).



### ***Costs Compared to Revenue: Single-family Construction***

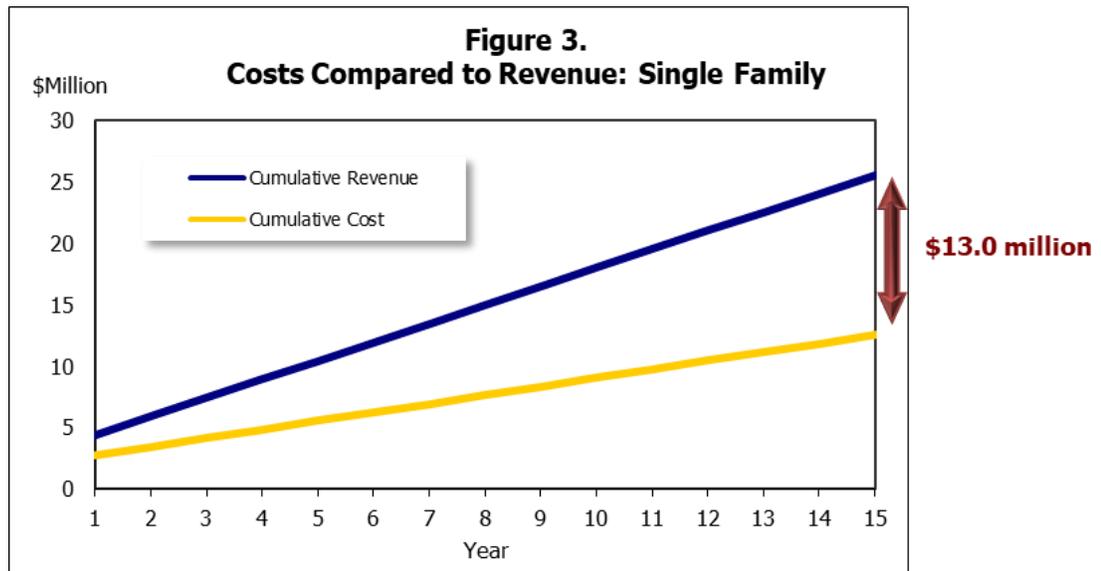
This section summarizes results for single-family construction only. The relevant assumptions about the single-family homes built (including their average price, property tax payments, and construction-related fees incurred) are described in the NAHB report, *The Metro Area Impact of Home Building in Milwaukee, Wisconsin: Income, Jobs and Taxes Generated*.

- In the first year, the 100 single-family homes built in Milwaukee result in an estimated
  - **\$4.4 million** in tax and other revenue for local governments,
  - **\$350,000** in current expenditures by local government to provide public services to the net new households at current levels, and
  - **\$2.3 million** in capital investment for new structures and equipment undertaken by local governments

The analysis assumes that local governments finance the capital investment by borrowing at the current municipal bond rate.

- In a typical year after the first, the 100 single-family homes result in
  - **\$1.5 million** in tax and other revenue for local governments, and
  - **\$699,000** in local government expenditures needed to continue providing services at current levels.

- The difference between government revenue and current expenditures is defined as an “operating surplus.” In this case, the operating surplus generated during the first year is large enough to service and pay off all debt incurred by investing in structures and equipment at the start of the first year by the end of the first year. After that, the operating surpluses will be available to finance other projects or reduce taxes. After 15 years, the homes will generate a cumulative **\$25.6 million in revenue** compared to **\$12.6 million in costs**, including annual current expenses, capital investment, and interest on debt (Figure 3).



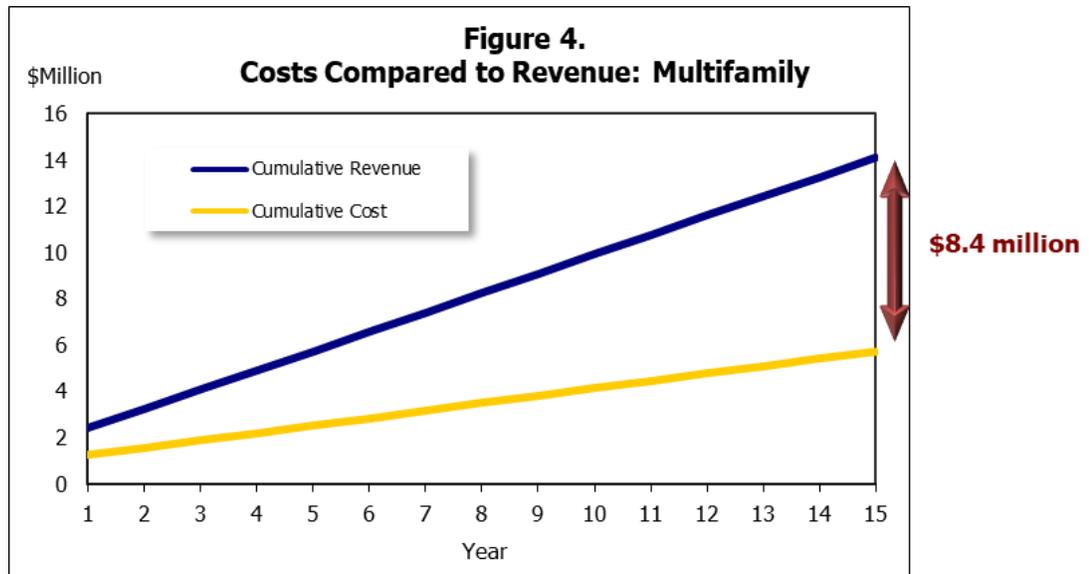
### ***Costs Compared to Revenue: Multifamily Construction***

This section summarizes results for multifamily construction only. As with the section on single-family construction, relevant assumptions about the units built can be found in *The Metro Area Impact of Home Building in Milwaukee, Wisconsin: Income, Jobs and Taxes Generated*.

- In the first year, the 70 multifamily housing units built in Milwaukee result in an estimated
  - **\$2.4 million** in tax and other revenue for local governments,
  - **\$160,000** in current expenditures by local government to provide public services to the net new households at current levels, and
  - **\$1.0 million** in capital investment for new structures and equipment undertaken by local governments

The analysis assumes that local governments finance the capital investment by borrowing at the current municipal bond rate.
- In a typical year after the first, the 70 multifamily units generate
  - **\$835,000** in tax and other revenue for local governments, and
  - **\$320,000** in local government expenditures needed to continue providing services at current levels.

- Again, the difference between government revenue and current expenditures is an “operating surplus.” As was the case for single-family construction, the first-year operating surplus generated by multifamily construction is large enough to service and pay off all debt incurred by investing in structures and equipment at the beginning of the first year by the end of the first year. After that, future operating surpluses will be available to finance other projects or reduce taxes. After 15 years, the multifamily homes will generate a cumulative **\$14.1 million in revenue** compared to **\$5.7 million in costs**, including annual current expenses, capital investment, and interest on debt (Figure 4).



### ***Method Used to Estimate Costs***

The method for estimating local government revenue generated by home building is explained in the attachment to *The Metro Area Impact of Home Building in Milwaukee, Wisconsin: Income, Jobs and Taxes Generated*. This section describes how costs are estimated.

The general approach is to assume local jurisdictions supply residents of new homes with the same services that they currently provide, on average, to occupants of existing structures. The amount that any jurisdiction spends is available from the Census of Governments, where all units of government in the U.S. report line item expenses, revenues, and intergovernmental transfers once every five years to the Governments Division of the U.S. Census Bureau. Census of Governments accounts can be aggregated for every local government in the Milwaukee-Waukesha metro area, and the result used to calculate total annual expenses per single-family and multifamily housing unit (Table 1).

Not surprisingly, cost per housing unit varies substantially across the major service categories. Education accounts for the largest share of annual expenses, followed at a distance by the shares for police protection, recreation and culture, and fire protection.

Table 1.  
Total Annual Local Government Expenses per Housing Unit

	Single-family	Multifamily
Education	\$3,105	\$1,831
Police Protection	\$1,091	\$793
Fire Protection	\$452	\$329
Corrections	\$350	\$254
Streets and Highways	\$141	\$81
Water Supply	\$217	\$127
Sewerage	\$204	\$119
Health Services	\$300	\$218
Recreation and Culture	\$542	\$394
Other General Government	\$396	\$288
Electric Utilities	\$62	\$45
Public Transit	\$133	\$97
<b>Total</b>	<b>\$6,993</b>	<b>\$4,577</b>

In deriving the above estimates, water supply and sewerage expenses are allocated based on gallons of water consumed per day by single-family and multifamily households. Streets and highway expenses are allocated based on average number of vehicle trips generated on weekdays. Education is allocated based on average number of public school children age 5 through 18. The remaining expenses listed in Table 1 are assumed to be proportional to household size and are allocated to single-family and multifamily units based on average number of persons per household.<sup>4</sup>

There are several factors present in most parts of the country that tend to reduce education expenses per housing unit. The first is the average number of children going to public schools present in the units. According to the American Community Survey, there is, on average, only a little over one public school child for every three households in the U.S. The number is about 0.4 per household for single-family and under 0.2 per household for multifamily. So education costs per housing unit are lower than costs per pupil, simply because there is less than one pupil per household.

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<sup>4</sup> Information about vehicle trips comes from *Trip Generation Manual, 10th Ed.*, September 2017, Institute of Transportation Engineers: <https://www.ite.org/tripgeneration/index.asp>. Information about water consumption comes from *Water Demand Trends in the Multifamily Housing Sector*, a study undertaken in 2017 by Jack Kiefer and Lisa Krentz for the Water Research Foundation <http://www.waterrf.org/Pages/Index3.aspx>. Information about household size and number of public school children comes from the 2016 Public Use Microdata Sample of the American Community Survey, U.S. Census Bureau: <https://www.census.gov/programs-surveys/acs/>.

Beyond that, state governments typically pay for some public school expenses in the form of intergovernmental transfers. In the latest Census of Governments, local governments in aggregate across the Milwaukee-Waukesha metro area spent about \$2.8 billion in current expenses on education. However, over half of this was offset by \$1.5 billion in state-to-local intergovernmental transfers for education.

In addition to current expenses, providing services to residents requires that local governments make capital expenditures for items such as schools and other buildings, equipment, roads, and other structures.

The process employed by NAHB to estimate capital costs involves several steps. The general approach is to apply parameters from a conventional economic model (a production relationship, where costs are expressed as a function of labor and capital) estimated with state level data to information for a specific local area. State and local government capital in each state can be derived through a procedure that has been established over several decades in the technical literature on public finance (see the technical appendix for details). The parameter estimates are then applied to a local area, where information is available for every variable except capital. The local capital stock then emerges as a residual in the calculation. Consistent with the approach used to estimate current expenses, the amount of capital in each category is expressed as the amount necessary to accommodate an average single-family or average multifamily housing unit (Table 2).

Table 2.  
Local Government Capital per Housing Unit

	Single-family	Multifamily
Schools	\$4,061	\$2,395
Hospitals	\$621	\$451
Other Buildings	\$1,777	\$1,292
Highways and streets	\$2,677	\$1,543
Conservation & development	\$540	\$393
Sewer systems	\$2,179	\$1,277
Water supply	\$5,544	\$3,250
Other structures	\$5,561	\$4,042
Equipment	\$173	\$126
Total	\$23,132	\$14,768

To implement these numbers, several conservative assumptions are made to avoid understating the costs. In contrast to the way current expenses were handled, intergovernmental transfers are generally not taken into account here—it is assumed that local governments undertake all capital investment without any help from the states. The exception is highways and streets, for which the amount of current expenditures per dollar of capital is typically quite low. It is further assumed that none of this demand for capital can be met through current excess capacity. Instead, local governments invest in new structures and equipment at the start of the first year, before any homes are built. To the extent that this is not true—that, for instance,

some revenue from impact or other fees is available to fund part of the capital expenditures—interest costs would be somewhat lower than reported here.

To compare the streams of costs and revenues over time, the analysis assumes that half of the current expenses and half of the ongoing, annual revenues are realized in the first year. This would be the case if construction and occupancy took place at an even rate throughout the year. Revenues in the first year also include all of the one-time construction impacts such as impact and permit fees.

The difference between revenues and current expenses in a given year is an operating surplus. At the start of the first year, capital investment is financed through debt by borrowing at the current municipal bond interest rate,<sup>5</sup> and the interest accrues throughout the year. Each year after that, the operating surplus is used first to pay the interest on the debt, if any exists, then to pay off the debt at the end of the year. Results for the 100 single-family homes are shown in Table 3, for the 70 multifamily units in Table 4, and for single-family and multifamily combined in Table 5.

The difference between revenues (the third column) and all costs, including interest on the debt, is shown in the last column. Again, the analysis assumes that any operating surplus is being used to service the debt, and then to retire as much debt as possible at the end of the year. For either single-family or multifamily construction considered in isolation, as well as for the more realistic combined scenario that analyzes both types of construction together, revenue net of costs and interest is negative in the first year but turns positive beginning in year two.

In fact, in all three cases (shown in Tables 3, 4 and 5) revenue net of costs and interest is sufficient to pay off all debt by the end of year one. After that, revenue net of costs generated by the 100 single-family and 70 multifamily homes is roughly \$1.3 million per year.

Net revenue for both structure types falls slightly in year 11, due to a cost that local governments incur at that time as capital equipment purchased at the start of the first year becomes fully depreciated and needs to be replaced. All other capital investment consists of structures of various types, and the effective service life for any type of structure is considerably longer than a single decade.

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<sup>5</sup>The interest rate on municipal bonds is the monthly Bond Buyer long-term Municipal Bond Index available on the Bond Buyer Web site: [https://data.bondbuyer.com/MarketStatisticsArchive/Search\\_MBI/11?Name=Municipal%20Bond%20Index](https://data.bondbuyer.com/MarketStatisticsArchive/Search_MBI/11?Name=Municipal%20Bond%20Index).

Table 3. Results for 100 Single-family Homes

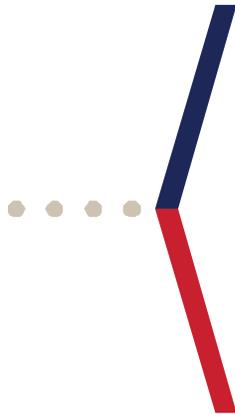
Year	Current Expenses	Revenue	Operating Surplus	Capital Investment Start of Year	Debt Outstanding End of Year	Interest on the Debt	Revenue Net of Costs and Interest
1	349,700	4,369,100	4,019,400	2,313,300	0	79,800	1,626,300
2	699,300	1,514,800	815,500	0	0	0	815,500
3	699,300	1,514,800	815,500	0	0	0	815,500
4	699,300	1,514,800	815,500	0	0	0	815,500
5	699,300	1,514,800	815,500	0	0	0	815,500
6	699,300	1,514,800	815,500	0	0	0	815,500
7	699,300	1,514,800	815,500	0	0	0	815,500
8	699,300	1,514,800	815,500	0	0	0	815,500
9	699,300	1,514,800	815,500	0	0	0	815,500
10	699,300	1,514,800	815,500	0	0	0	815,500
11	699,300	1,514,800	815,500	17,300	0	0	798,200
12	699,300	1,514,800	815,500	0	0	0	815,500
13	699,300	1,514,800	815,500	0	0	0	815,500
14	699,300	1,514,800	815,500	0	0	0	815,500
15	699,300	1,514,800	815,500	0	0	0	815,500

Table 4. Results for 70 Multifamily Homes

Year	Current Expenses	Revenue	Operating Surplus	Capital Investment Start of Year	Debt Outstanding End of Year	Interest on the Debt	Revenue Net of Costs and Interest
1	160,300	2,385,800	2,225,500	1,033,800	0	35,700	1,156,000
2	320,500	835,000	514,500	0	0	0	514,500
3	320,500	835,000	514,500	0	0	0	514,500
4	320,500	835,000	514,500	0	0	0	514,500
5	320,500	835,000	514,500	0	0	0	514,500
6	320,500	835,000	514,500	0	0	0	514,500
7	320,500	835,000	514,500	0	0	0	514,500
8	320,500	835,000	514,500	0	0	0	514,500
9	320,500	835,000	514,500	0	0	0	514,500
10	320,500	835,000	514,500	0	0	0	514,500
11	320,500	835,000	514,500	8,800	0	0	505,700
12	320,500	835,000	514,500	0	0	0	514,500
13	320,500	835,000	514,500	0	0	0	514,500
14	320,500	835,000	514,500	0	0	0	514,500
15	320,500	835,000	514,500	0	0	0	514,500

Table 5. Combined Results for 100 Single-family and 70 Multifamily Homes

Year	Current Expenses	Revenue	Operating Surplus	Capital Investment Start of Year	Debt Outstanding End of Year	Interest on the Debt	Revenue Net of Costs and Interest
1	510,000	6,754,900	6,244,900	3,347,100	0	115,500	2,782,300
2	1,019,800	2,349,800	1,330,000	0	0	0	1,330,000
3	1,019,800	2,349,800	1,330,000	0	0	0	1,330,000
4	1,019,800	2,349,800	1,330,000	0	0	0	1,330,000
5	1,019,800	2,349,800	1,330,000	0	0	0	1,330,000
6	1,019,800	2,349,800	1,330,000	0	0	0	1,330,000
7	1,019,800	2,349,800	1,330,000	0	0	0	1,330,000
8	1,019,800	2,349,800	1,330,000	0	0	0	1,330,000
9	1,019,800	2,349,800	1,330,000	0	0	0	1,330,000
10	1,019,800	2,349,800	1,330,000	0	0	0	1,330,000
11	1,019,800	2,349,800	1,330,000	26,100	0	0	1,303,900
12	1,019,800	2,349,800	1,330,000	0	0	0	1,330,000
13	1,019,800	2,349,800	1,330,000	0	0	0	1,330,000
14	1,019,800	2,349,800	1,330,000	0	0	0	1,330,000
15	1,019,800	2,349,800	1,330,000	0	0	0	1,330,000



# **Comparing Costs to Revenue for Local Governments**

## **Technical Appendix on Estimating Capital Owned and Maintained by Local Governments**

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## Technical Appendix on Estimating Local Capital Owned and Maintained by Local Governments

This appendix explains the method used to estimate the age and dollar value of local government capital by function (education, water and sewer services, etc.). The general approach is to estimate economic relationships using state-level data and then apply parameters from the state-level estimates to local data.

First, a cost share equation based on conventional production theory is described for the structures associated with each function of government. In the equations age of capital is used as a proxy for technologic change. Age of capital, in turn, is estimated as a function of population growth.

The following derivations apply to any one of the ten categories of state and local government capital—e.g., highways or school buildings—tracked in the Bureau of Economic Analysis (BEA) wealth data files. For simplicity, the notation suppresses an explicit reference to capital type. In cases where some detail of the model pertains to a particular type of capital or function of local governments, the text will make that clear.

Let  $y$  = output;  $L$  = labor,  $w$  = the price of labor, and  $r$  = the price of capital, and consider a general translog cost function:<sup>6</sup>

$$(1) \quad c_{it} = \beta_0 + \beta_w \ln w_{it} + \beta_r \ln r_{it} + \beta_y \ln y_{it} + \beta_a a_{it} + \frac{1}{2} \beta_{ww} (\ln w_{it})^2 + \beta_{wr} \ln w_{it} \ln r_{it} \\ + \frac{1}{2} \beta_{rr} (\ln r_{it})^2 + \beta_{wy} \ln w_{it} \ln y_{it} + \beta_{ry} \ln r_{it} \ln y_{it} + \beta_{wa} a_{it} \ln w_{it} + \beta_{ra} a_{it} \ln r_{it} \\ + \beta_{yy} (\ln y_{it})^2 + \beta_{ya} a_{it} \ln y_{it} + \beta_{aa} a_{it}^2$$

In the case where the firm is a government,  $y_{it}$  is essentially unmeasurable, so it seems reasonable to assume linear homogeneity in output. This simplifies the translog specification considerably:

$$(2) \quad c_{it} = \beta_0 + \beta_w \ln w_{it} + \beta_r \ln r_{it} + \ln y_{it} + \beta_a a_{it} + \frac{1}{2} \beta_{ww} (\ln w_{it})^2 + \beta_{wr} \ln w_{it} \ln r_{it} \\ + \frac{1}{2} \beta_{rr} (\ln r_{it})^2 + \beta_{wa} a_{it} \ln w_{it} + \beta_{ra} a_{it} \ln r_{it} + \beta_{aa} a_{it}^2$$

Specification (2) still requires an estimate of  $\ln y_{it}$ . However, application of Shephard's Lemma generates the following two-equation system:

$$(3) \quad s_{L, it} = w_{it} L_{it} / c_{it} = \partial \ln c_{it} / \partial \ln w_{it} = \beta_w + \beta_{ww} \ln w_{it} + \beta_{wr} \ln r_{it} + \beta_{wa} a_{it} \\ (4) \quad s_{K, it} = r_{it} k_{it} / c_{it} = \partial \ln c_{it} / \partial \ln r_{it} = \beta_r + \beta_{wr} \ln w_{it} + \beta_{rr} \ln r_{it} + \beta_{ra} a_{it}$$

By estimating cost shares rather than the cost function itself, the ability to estimate  $\beta_0$ ,  $\beta_a$ , and  $\beta_{aa}$  (essentially nuisance parameters) is lost. Also lost is some precision, in the sense that a lower-order approximation is being estimated.<sup>7</sup> The advantage is relief from the need to supply values for the unobservable  $y_{it}$ .

<sup>6</sup> See, for example, Walter Diewert and Terry Wales (1987), "Flexible Functional Forms and Global Curvature Conditions," *Econometrica*, 55, 43-68.

<sup>7</sup> See Henri Theil, *The System-Wide Approach to Microeconomics*, University of Chicago Press, 1980, page 151.

Economic theory implies several restrictions.

Symmetry:  $\beta_{wr}$  is the same in both equations

Linear homogeneity in input prices:  $\beta_w + \beta_r = 1$ ;  $1/2 \beta_{ww} + \beta_{wr} + 1/2 \beta_{rr} = 0$ ;  $\beta_{wa} + \beta_{ra} = 0$ .

The restrictions are imposed in the usual way. One of the factor prices ( $w_{it}$ ) is used as a numeraire; and only one share equation ( $s_{L, it}$ ) is estimated, leaving parameters of the second, if needed, to be recovered by simple algebra. The resulting estimating equation is

$$(5) \quad s_{L, it} = w_{it} L_{it} / (w_{it} L_{it} + r_{it} k_{it}) = \beta_w + \beta_{wr} \ln (r_{it} / w_{it}) + \beta_{wa} a_{it} + \beta_I' I_{it}$$

where  $I_{it}$  is a vector of indicator variables that may be added to equations for some government functions to account for outliers among specific states and time periods. More detail is provided when the regression results are discussed.

Model (5) can be estimated with any standard regression package, provided state-level annual data for  $L$ ,  $w$ , and  $r$  can be specified. Series beginning in 1987 for the first two are available from the Government Division of the U.S. Census Bureau. For  $r$ , standard practice is followed by assuming cost of capital is the sum of three terms: maintenance (meaning, in this case, all non-labor operating costs), interest, and depreciation.

$$(6) \quad r_{it} = x_{it} / k_{it} + \phi_{it} + \xi_t$$

where  $x_{it}$  is the difference between total current expenditures and labor costs,  $\phi_{it}$  is an interest rate for appropriate types of tax-exempt public-purpose government bonds, and  $\xi_t$  is the national depreciation rate from BEA's wealth accounts.

To estimate the cost share equations, the same annual interest rate series  $\phi_t$  is used for all states. Because the preferred series not available until 1990, two different sources are used to construct the 1987–2001 annual interest rate series  $\phi_t$ . From 1987 through to the end of 1989, the JP Morgan Revenue Bond Index (RBI) is used. The JP Morgan RBI data are monthly. An annual interest rate is constructed by taking the average of the 12 monthly observations for each calendar year.

From 1990 to the present the Merrill Lynch 20 Year AAA GO series is used. The Merrill Lynch data are provided weekly. An annual interest rate is constructed by taking the average of the 52 observations in each calendar year.

To insure that there is no discontinuity in the series, the annual interest rate from the JP Morgan RBI index for the years 1987 1988 and 1989 is multiplied by the average of the annual ratio of the Merrill Lynch 20 Year AAA GO series divided by the JP Morgan RBI index the for the years 1990 to the present. That ratio turned out to be 0.93. The reason the ratio is less than one is largely because the Merrill Lynch index has a duration that is on average 5 years shorter than the JP Morgan RBI Index.

The final index was chosen following consultation with bonds specialists at both JP Morgan and Merrill Lynch. Although there are hundreds of thousands of unique muni-bonds, and most are rarely if ever traded, the experts felt that a 20 year maturity seemed appropriate and that the ML GO AAA series was probably best for this purpose.

In order to make the cost share equations operational, it's necessary to apportion equipment among the other nine types of capital for which it's possible to approximately match capital with expense and employment data by function of government. In general, a year-zero approach is employed, basing the analysis on the ratio of structures to equipment when both are brand new.

Suppressing the cross-sectional (state) subscript, capital  $k$  required for a specific local government function is the sum of structures  $k_s$  and equipment  $k_e$ :

$$(7) \quad k_t = k_{st} + k_{et}$$

where  $k_{st} = k_{s0}(1-\xi_s)^{a_s}$ ,  $k_{et} = k_{e0}(1-\xi_e)^{a_e}$

or, equivalently,

$$(8) \quad k_{s0} = k_{st}(1-\xi_s)^{-a_s}, \quad k_{e0} = k_{et}(1-\xi_e)^{-a_e}$$

Brand new equipment is allocated to brand new structures based on the relative total year-zero values of structures. From this, a ratio  $z$  can be derived, which will be the same for all local government functions (or structure types):

$$(9) \quad z = k_{e0}/k_{s0} = k_{et}(1-\xi_e)^{-a_e} k_{st}^{-1}(1-\xi_s)^{a_s}$$

The average  $z$  ratio for 50 states plus the District of Columbia in the most recent year for which we can compute it (1998) is .11642. This number is used below to help derive estimates of government-owned equipment and structures for a particular local area.

The blended ages and depreciation rates for total capital (structures and equipment) were used to compute the independent variables in the estimating equations. The nine equations (one for each function of government) were estimated, using data for the period where complete state-level government employment and finance data were available—1987 through 1998. The procedure converged quickly (in four iterations). Results are shown in Table 3.

Fit of the model was improved by including a number of indicator variables, up to three per equation. These are identified as I1, I2, and I3 in Table A1 and defined in Table A2.

Not all of the cost equations contain an indicator variable, and each indicator captures only a small number of states. Several variables simply indicate that an observation is for the state of Alaska, and it seems reasonable to suppose that the technology of providing some government services in Alaska would be different than in many other states. In the case of housing, New York appears to be an isolated outlier, and again that is not especially surprising. Other indicators capture a small number of states in New England or the Rocky Mountain area. The conservation series showed a clear break between 1991 and 1992 in Arizona. The Census Bureau instituted some procedural changes involving the collection and reporting of government finance data beginning in 1992.

**Table A1. Regression Results: Cost Share Equations**

	$\beta_w$	$\beta_{wr}$	$\beta_{wa}$	I1	I2	I3	Adj R <sup>2</sup>
Residential	-0.5454 (.0001)	-0.1082 (.0001)	0.0051 (.0158)	0.1531 (.0001)	0.2150 (.0001)		.453
Education	-0.3801 (.0001)	-0.1391 (.0001)	0.0156 (.0001)				.545
Hospital	0.5682 (.0001)	-0.1413 (.0001)	-0.0247 (.0001)	-0.1793 (.0001)			.506
Other Buildings	0.3970 (.0001)	-0.1655 (.0001)	-0.0368 (.0001)				.784
Streets & Highways	-0.0345 (.4529)	-0.0723 (.0001)	-0.0110 (.0001)	0.2072 (.0001)			.598
Conservation	0.1846 (.0165)	-0.0524 (.0001)	-0.0017 (.6021)	0.3443 (.0001)	-0.2017 (.0001)	0.1210 (.0001)	.483
Sewer	-0.4148 (.0001)	-0.0861 (.0001)	0.0018 (.1985)				.522
Water	-0.0336 (.5780)	-0.1077 (.0001)	-0.0169 (.0001)				.413
Other Structures	-0.2342 (.0021)	-0.1112 (.0001)	-0.0111 (.0004)	0.39629 (.0001)			.566

**Table A2: Indicator Variables for Cost Share Equations**

Capital type	Variable	Condition for I=1
Residential	I1	state=AK
	I2	state=NY
Hospital	I1	state=AZ, NH, or VT
Streets & Highways	I1	state=AK
Conservation	I1	state=AK
	I2	state =NY or CT; or state=AZ and year < 1992
	I3	state=ID, MT, ND, or WY
Other Structures	I1	state= NE, NY, or WA

In the equations above, age of the capital stock appears as an explanatory variable. This is not readily available, even at the state level. A commonly used approach employs perpetual accounting, investment, and depreciation rates to base-year estimates.<sup>8</sup> The procedure used here begins with that approach, but then relates the investment rates to population growth rates, one of the few items for which consistent time series are available for individual U.S. counties.

From BEA national wealth data, the following are available or can easily be computed:

$\xi$  = real annual rate of depreciation (defined broadly, as BEA does, to include a normal rate of obsolescence and retirement of assets)

$\delta$  = monthly depreciation rate, a simple algebraic transformation of  $\xi$

$N_t$  = real, net (of depreciation) rate of investment in year  $t$ ,  $t=1946, \dots, 2000$ .

<sup>8</sup> As in Douglas Holtz-Eakin, "State-Specific Estimates of State and Local Government Capital," *Regional Science and Urban Economics*, Vol. 23, No. 2, April 1993, pp. 185-210.

From data compiled by the Governments Division of the Census Bureau, and ratios employed by BEA to analyze this data, the following can be computed for state  $i$  and  $t=1977, \dots, 1999$ :

$vn_{it}$  = real investment in new assets state  $i$  in year  $t$ .

$ve_{it}$  = real investment in existing assets state  $i$  in year  $t$ .

$v_{it}$  = real investment in state  $i$  in year  $t = vn_{it} + ve_{it}$ .

$x_{it}$  = current expenditures associated with the relevant type of capital state  $i$  in year  $t$ .

From standard Census Bureau data it is possible to compute

$\Pi_{it}$  = population growth in the state relative to the national rate; i.e.,

$$\Pi_{it} = \frac{\Delta \rho_{it}}{\rho_{it-1}} \left[ \frac{\sum_i \Delta \rho_{it}}{\sum_i \rho_{it-1}} \right]^{-1}$$

The starting point consists of initial end-of-year estimates of the real capital stock,  $k_{i76}^0$ , determined by allocating capital to each state according to its share of current expenditure,  $x_{i77}$ . This procedure, the one employed for example by Holtz-Eakin (1993), is used here only for the purpose of supplying initial values to be modified in subsequent iterations.

Perpetual inventory accounting can be used to calculate the following recursively for  $t=1977, \dots, 1999$ :

$$(10) \quad k_{i,t+1}^0 = k_{it}^0 (1-\xi) + v_{it+1} (1-\delta)^6$$

This assumes that investment made during period  $t+1$  depreciates an average of 6 months by the end of the period. Then relative (to the national rate) net real rates of investment can also be computed:

$$(11) \quad \equiv_{it} = \left[ \frac{v_{it} - \delta k_{it-1}^0}{k_{it-1}^0} \right] N_t^{-1}$$

The goal is to obtain estimates of parameters  $\forall_j$  and  $2_q$  in the following regression relationship:

$$(12) \quad \equiv_{it} = \sum_{j=1}^J \alpha_j^0 \rho_{it-j}^0 + \sum_{q=1}^Q \mathcal{G}_q D_q$$

where  $J$  is the longest lag considered and the  $D_q$  are indicator (dummy) variables. The hypothesis underlying this specification is that a state's rate of investment (relative to the national rate) is a function of past rates of its population growth (also relative to the national rate), with indicator variables to account for anomalies in some states due to peculiarities that are difficult to observe and quantify. Inspection of the pair wise correlations between  $\equiv_{it}$  and  $\Pi_{it-j}$  reveal that they begin to decline at or before the lag reaches eight years, depending on the type of capital. Thus, model specification for each type of capital began by tentatively considering population growth effects up to  $J=8$ . The final specification varies from case to case.

As a practical matter, the final specifications employ averages of population growth rates lagged over several years. Over the course of several experiments, the sum of the coefficients on the population variables never changed substantially when an average was substituted for a series of individual lags. Coefficients on individual lags tended to fluctuate widely and lack statistical significance, due to collinearity. The use of averages thus aids interpretation without impacting the marginal impacts predicted by the equations in a meaningful way.

Three indicator variables were used in all but the hospital capital equation, which employed four. In most cases, indicator variables flag relatively few states (Table A3).

**Table A3: Indicator Variables for Relative Investment Rate Equations**

Capital Category	DVERYHI=1	DHIGH=1	DLOW=1	DVERYLOW=1
1 Equipment	DC, WY	AZ, CO, MT, UT	AR, NH, RI	
2 Residential Buildings	DC, HI, MA, NY	CT, DE, RI	CO, FL, ID, NM, TX, UT, VT, WY	
3 Educational Buildings	WY	HI, NM, TX	CA, VT, WI	
4 Hospital Buildings	WY	AL, FL, GA, HI, IA, ID, KS, NY, OH, WA	AR, CT, DE, IL, KY, ME, OR, UT, WI, WV	AZ, VT
5 Other Buildings	DC, WY	HI, MD	AR	
6 Highways and Streets	WY	DC, IA, MN, MT, ND, NE	AR, ME, NH, SC, VT	
7 Conservation & Development	HI, WY	AZ, LA, MT	AL, NY, OK, TN, VA	
8 Sewer Systems & Structures	DC, NY, WA	MA, MD, NJ, OH, RI, WI	AR, NC	
9 Water Supply Facilities	CO, DC, SD, WY	FL, NV	DE, NH	
10 Other Structures	DC	NE	NH	

Given initial estimates, it's possible to begin the perpetual inventory accounting process at an earlier date. If we assume that the World War II period was atypical and restrict ourselves to post-war population data, an 8-year lag in (12) implies that 1954 is the first year for which we can obtain state investment estimates. Hence, state capital stocks in 1953 are estimated by allocating the national capital stock in that year according to its share of the U.S. population, then estimating state investment in the years from 1954 through 1976 recursively according to

$$(13) \quad v_{it}^0 = k_{it-1}^0 (\xi + N_t \equiv_{it}^0)$$

where  $\equiv_{it}^0$  is estimated from (12). In words, (13) says that investment is enough to cover depreciation, plus another term which is the net national rate of investment multiplied by a relative factor specific to state *i*. It is then possible to combine (13) with (10) to derive estimates of the capital stock for the years 1954 through 1976 in most states. (Lack of complete data for in earlier years pushes the first estimate for Alaska forward to 1962.)

In this way revised estimates  $k_{i76}^1$  are derived, and these can be used to restart the process by repeating steps (10) through (13). This results in successively revised estimates  $k_{it}^1$  and  $\pi_{it}^1$  for  $t=1977, \dots, 1999$ ; parameters  $\nu_j^1$  and  $z_{jt}^1$ ;  $\nu_{it}^1$  for  $t=54, \dots, 76$ ; and  $k_{i76}^2$ . This ends the first iteration.

This process can be repeated until either a convergence criterion is satisfied. The particular criterion used was an average absolute percentage change in the  $k_{i76}$  no greater than  $10^{-10}$  between iterations.

The procedure was carried out for all 10 BEA categories of state and local government capital. Each of the ten equations converged in fewer than 10 iterations. The final estimates are shown in Table A4.

**Table A4. Final Regression Results: Dependent Variable=Relative Investment Rate**

	Equipment	Residential	Education	Hospital	Buildings nec
Iterations to Convergence	8	6	6	6	6
<b>Final Regression Coefficients (p-values):</b>					
Constant	-0.2590 (.0003)	0.5460 (.0001)	-0.0227 (.8295)	0.3663 (.0001)	0.5439 (.0001)
<i>Lagged relative population growth rates:</i>					
Population lag 1	0.4337 (.0001)		0.3852 (.0001)		0.1336 (.0001)
Population lag 2-5	0.1707 0.0212	0.0662 (.1225)			
Population lag 2-8			0.6865 (.0001)		0.0961 (.0002)
Population lag 6-8		0.0805 (.0532)		0.1270 (.0009)	
<i>State indicator variables:</i>					
DVeryhi	5.6639 (.0001)	2.9842 (.0001)	7.2485 (.0001)	4.1282 (.0001)	1.7082 (.0001)
DHigh	1.2733 (.0002)	0.7862 (.0001)	1.6538 (.0001)	1.4240 (.0001)	1.3839 (.0001)
DLow	-1.3392 (.0001)	-0.8119 (.0001)	-1.2254 (.0003)	-0.8407 (.0001)	-0.6383 (.0001)
DVerylow				-1.7778 (.0001)	
Adjusted R <sup>2</sup>	.432	.426	.311	.323	.402

**Table A4. Continued**

	Streets	C&D	Sewer	Water	Other
Iterations to Convergence	6	6	6	6	8
<b>Final Regression Coefficients (p-values):</b>					
Constant	0.8370 (.0001)	0.0938 (.0617)	0.4386 (.0001)	0.2036 (.0001)	0.2754 (.0016)
<i>Lagged relative population growth rates:</i>					
Population lag 1				0.1967 (.0001)	0.2253 (.0030)
Population lag 2		0.0950 (.0371)			
Population lag 2-5	0.2462 (.0001)				
Population lag 5			0.0516 (.1461)		
Population lag 2-8				0.4270 (.0001)	0.5368 (.0001)
Population lag 3-8		0.2653 (.0001)			
Population lag 6-8	0.0770 (.0318)		0.0701 (.0594)		
<i>State indicator variables:</i>					
DVeryhi	4.955 (.0001)	2.387 (.0001)	1.348 (.0001)	2.270 (.0001)	13.405 (.0001)
DHigh	1.340 (.0001)	1.223 (.0001)	1.025 (.0001)	0.396 (.0206)	5.981 (.0001)
DLow	-0.684 (.0006)	-0.785 (.0001)	-0.745 (.0001)	-0.126 (.0001)	-2.172 (.0001)
Adjusted R <sup>2</sup>	.502	.338	.268	.496	.528

The estimated pre-1977 investment series can be spliced onto the 1977-1999 data and the results used to estimate the average age of capital, by type, in each state. The procedure is as follows. First, set the average age of capital in state equal to the national average for 1953. Then, use perpetual accounting to recursively calculate the average age in subsequent years:

$$(14) \quad a_{it+1} = [(a_{it} + 1) k_{it}(1-\xi) + \frac{1}{2} v n_{it+1}(1-\rightarrow)^6 + ap_t ve_{it+1}(1-\rightarrow)^6] / k_{it+1}^0$$

where  $ap_t$  is the average age of the relevant type of private capital, in accord with the method used by BEA which assumes that existing assets purchased by governments are "typical".

The process of deriving estimating capital stock estimates for a particular local area begins by adapting the average age equation (14) to location m:

$$a_{mt} = [(a_{mt-1} + 1) k_{mt-1}(1-\xi) + g_t v_{mt}(1-\rightarrow)^6] / [k_{mt-1}(1-\xi) + v_{mt}(1-\rightarrow)^6]$$

where  $g_t = \frac{.5 \sum_i v n_{it} + pa \sum_i v e_{it}}{\sum_i v_{it}}$ , that is, the average end-of-the year age of total assets (including both new and used) purchased by all states in the country during the period.

Then (13) is substituted into the average age formula and the capital factor is eliminated in order to obtain

$$(15) \quad a_{mt} = \frac{(a_{mt-1} + 1)(1 - \delta) + g_t (\delta + N_t \eta_{mt})(1 - \varepsilon)^6}{1 - \delta + (\delta + N_t \eta_{mt})(1 - \varepsilon)^6}$$

Equation (13) can be used to estimate  $\bar{m}_t$  from local relative population growth factors  $\Pi_{mt}$ . Starting with the national average age for 1954 as initial estimate of the average age of the capital stock in  $m$ , (15) can be applied to calculate  $a_{mt}$  recursively for subsequent years.

The result is a recipe for estimating the age of the capital stock for a particular local area. To be implemented, the recipe requires only data on local population growth.

Given the age estimate—along with estimates of the parameters  $\beta_w$ ,  $\beta_{wr}$ , and  $\beta_{wa}$  from the cost share equations, capital depreciation rates  $\xi_t$  from BEA, a current rate on tax-exempt bonds  $\phi_{mt}$ , and values for  $w_{mt}$ ,  $L_{mt}$ , and  $x_{mt}$  that can be obtained for any unit of government from data bases maintained by the U.S. Census Bureau—capital  $k_{mt}$  is the only unknown in the local cost share equation

$$(16) \quad [w_{mt} L_{mt} + x_{mt} + (\phi_{mt} + \xi_t) k_{mt}] \cdot [\beta_w + \beta_{wr} \ln((x_{mt}/k_{mt} + \phi_{mt} + \xi_t)/w_{mt}) + \beta_{wa} a_{mt} + \beta'_t I_{mt}] = w_{mt} L_{mt}$$

However, it's necessary to account for the fact that capital in (16) consists of both structures and equipment. Equations (7), (8), and (9) imply that

$$(17) \quad k_{mt,s} = \gamma_{mt} k_{mt} \text{ and } k_{mt,e} = (1 - \gamma_{mt}) k_{mt} \text{ where}$$

$$(18) \quad \gamma_{mt} = [1 + z(1 - \xi_e) a_{mt,e} (1 - \xi_s)^{-a_{mt,s}}]^{-1}$$

By using the 1998 state average value (.11642) for  $z$ , it's possible to compute  $\gamma_{mt}$  from BEA's depreciation rates and the estimated ages of structures and equipment. In turn,  $\gamma_{mt}$  can be used to compute

$$(19) \quad a_{mt} = a_{mt,s} k_{mt,s} / k_{mt} + a_{mt,e} k_{mt,e} / k_{mt} = \gamma_{mt} a_{mt,s} + (1 - \gamma_{mt}) a_{mt,e}$$

and

$$(20) \quad \xi_{mt} = \gamma_{mt} \xi_{t,s} + (1 - \gamma_{mt}) \xi_{t,e}$$

for the blended age and depreciation rate of capital, respectively. Substitution into (16) yields a formula that can be applied in practice:

$$(21) \quad [w_{mt} L_{mt} + x_{mt} + (\phi_{mt} + \gamma_{mt} \xi_{t,s} + (1 - \gamma_{mt}) \xi_{t,e}) k_{mt}] \cdot [\beta_w + \beta_{wr} \ln((x_{mt}/k_{mt} + \phi_{mt} + \gamma_{mt} \xi_{t,s} + (1 - \gamma_{mt}) \xi_{t,e})/w_{mt})] + \beta_{wa} (\gamma_{mt} a_{mt,s} + (1 - \gamma_{mt}) a_{mt,e}) + \beta'_t I_{mt}] = w_{mt} L_{mt}$$

This is the formula used to estimate  $k_{mt}$ , the dollar value of a particular type of government capital in a particular local area. Because capital appears twice in the nonlinear expression, a closed form solution for it does not exist. Finding the solution is a one-dimensional problem, however, so  $k_{mt}$  can be recovered through elementary numerical methods.